BLUE PRINT - III
MATHEMATICS - XII

S.No.	Topic	VSA	SA	LA	TOTAL
1. (a)	Relations and Functions	1 (1)	4 (1)	-	10 (4)
(b)	Inverse trigonometric Functions	1 (1)	4 (1)	-	
2. (a)	Matrices	2 (2)	-	6 (1)	-
(b)	Determinants	1 (1)	4 (1)	-	13 (5)
3. (a)	Continuity and Differentiability	-	12 (3)	-	}18 (4)
(b)	Applications of Derivatives	-	-	6 (1)	44 (11)
(c)	Integrals	2 (2)	4 (1)	6 (1)	
(d)	Applications of Integrals			6 (1)	}18 (5)
(e)	Differential equations		8 (2)		8 (2)
4. (a)	Vectors	3 (3)	4 (1)	-	
(b)	3 - dimensional geometry	-	4 (1)	6 (1)	17 (6)
5.	Linear - Programming	-	-	6 (1)	6 (1)
6.	Probability	-	4 (1)	6 (1)	10 (2)
	Total	10 (10)	48 (12)	42 (7)	100 (29)

Sample Question Paper - III

Time : 3 Hours **Max. Marks : 100**

General Instructions :

1. All questions are compulsory.

2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 mark each, section B is of 12 questions of 4 marks each and section C is of 7 questions of 6 marks each.

3. There is no overall choice. However, an internal choice has been provided in four questions of 4 marks each and two questions of six marks each.

4. Use of calculators is not permitted. However, you may ask for Mathematical tables.

SECTION - A

1. Let $f : R - \left\{-\frac{3}{5}\right\} \to R$ be a function defined as $f(x) = \frac{2x}{5x+3}$, find f^{-1} : Range of $f \to R - \left\{-\frac{3}{5}\right\}$

2. Write the range of one branch of $\sin^{-1}x$, other than the Principal Branch.

3. If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$, find x, $0 < x < \frac{\pi}{2}$ when $A + A' = I$

4. If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or skew symmetric.

5. On expanding by first row, the value of a third order determinant is $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$. Write the expression for its value on expanding by 2nd column. Where A_{ij} is the cofactor of element a_{ij}.

6. Write a value of $\int \frac{1+\cot x}{x + \log \sin x}\, dx$

7. Write the value of $\int_0^{\pi/2} \log\left[\frac{3+5\cos x}{3+5\sin x}\right] dx$

8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ and $\vec{b}$?

9. Write the value of $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i}) + \hat{k}.(\hat{j}\times\hat{i})$

10. For two non zero vectors $\vec{a}$ and $\vec{b}$ write when $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$ holds.

SECTION - B

11. Show that the relation R in the set $A = \{x \mid x \in W, 0 \le x \le 12\}$ given by $R = \{(a, b) : (a - b) \text{ is a multiple of } 4\}$ is an equivalence relation. Also find the set of all elements related to 2.

OR

Let * be a binary operation defined on N × N, by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Also find the identity element for * on N × N, if any.

12. Solve for x :

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}, \ |x| < 1$$

13. If a, b and c are real numbers and

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Show that either a + b + c = 0 or a = b = c.

14. If $f(x) = \begin{cases} \dfrac{x-5}{|x-5|} + a, & \text{if } x < 5 \\ a + b, & \text{if } x = 5 \\ \dfrac{x-5}{|x-5|} + b, & \text{if } x < 5 \end{cases}$

is a continuous function. Find a, b.

15. If $x^y + y^x = \log a$, find $\dfrac{dy}{dx}$.

16. Use lagrange's Mean Value theorem to determine a point P on the curve $y = \sqrt{x-2}$ where the tangent is parallel to the chord joining (2, 0) and (3, 1).

17. Evaluate :

$$\int \frac{1}{\cos(x-a)\cos(x-b)}\, dx$$

OR

Evaluate :

$$\int \frac{2+\sin x}{1+\cos x} \cdot e^{x/2} \cdot dx.$$

18. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$

OR

If are the dia gonals of a parallelogram with sides , $\vec{a}$ and $\vec{b}$ ind the area of parallelogram in terms of and hence find the area with $\vec{d_1} = i + 2\hat{j} + 3\hat{k}$ and $\vec{d_2} = 3i - 2\hat{j} + k$.

19. Find the shortest distance between the lines, whose equations are

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{10-z}{-7} \text{ and } \frac{x-15}{3} = \frac{58-2y}{-16} = \frac{z-5}{-5}.$$

20. A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of the number appearing on the tickets $(x_1 < x_2 < x_3 < x_4 < x_5)$. Find the probability that $x_3 = 30$.

21. Show that the differential equation

$$2y\, e^{x/y} dx + \left(y - 2x\, e^{x/y}\right) dy = 0 \text{ is}$$

homogeneous and find its particular solution given that $x = 0$ when $y = 1$.

OR

Find the particular solution of the differential equation $\frac{dx}{dy} + y \cot x = 2x + x^2 \cot x,\ \ x \neq 0$

given that y = 0, when $x = \frac{\pi}{2}$

22. Form the differential equation representing the family of ellipses having foci on x-axis and centre at origin.

<u>SECTION - C</u>

23. A letter is known to have come from either TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probabiity that the letter has come from

(1) Tata nagar (ii) Calcutta

OR

Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distribution.

24. Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

25. Using integration, compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$

OR

Find the ratio of the areas into which curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$.

26. Evaluate :

27. A point on the hypotenuse of a right triangle is at a distance ‘a’ and ‘b’ from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left[a^{2/3}+b^{2/3}\right]^{\frac{3}{2}}$.

28. Using elementary transformations, find the inverse of the matrix

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$$

29. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.

$$\int \frac{e^{\tan^{-1} x}}{\left(1+x^2\right)^2}\, dx.$$